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LAMINAR FLOW HEAT TRANSFER AND PRESSURE DROP WITH FREEZING AT THE WALL

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NOMENCLATURE

c, specific heat;
D, inside tube diameter;
g, acceleration due to gravity;
Gr, Grashof number, $g\beta\Delta T D^3/v^2$;
Gz, $Re\,Pr\,D/L$;
h, heat transfer coefficient based on arithmetic mean temperature difference;
k, thermal conductivity;
L, test section length;
m, mass flow rate;
Nu, Nusselt number, hD/k;
p, pressure;
p^*, dimensionless pressure drop, $2(p_0 - p)/\rho V^2$;
Pr, Prandtl number, $c\mu/k$;
q, heat transfer rate;
q^*, dimensionless heat transfer rate, $q/mc(T_0 - T_f)$;
r, radius of liquid–solid interface;
R, tube radius;
Re, Reynolds number, $4m/\pi D\mu$;
T, temperature;
$\bar{T}_b$, average bulk fluid temperature;
T_w^*, $\dfrac{k_s(T_f - T_w)}{k_L(T_0 - T_f)}$;
V, average velocity at inlet;
Z^*, $4L/Re\,Pr\,D$;
β, coefficient of thermal expansion;
μ, dynamic viscosity;
v, kinematic viscosity;
ρ, fluid density.

Subscripts
b, bulk;
f, freezing point;
o, inlet;
w, wall;
s, solid;
L, liquid.

INTRODUCTION

WHEN the medium surrounding a pipe in which liquid is flowing is at a temperature below the freezing point of the fluid, the possibility of solidification at the wall exists. In many cases, freezing may be avoided by proper design of insulation or control of fluid flow rate and other system variables, but if none of these is possible, it becomes necessary to consider the effect of the solid layer on heat transfer and pressure drop.

The literature reveals three previous investigations dealing with this problem: Brush [1], Hirschberg [2] and Zerkle and Sunderland [3]. Zerkle and Sunderland's work is the most recent, and Ref. [3] should be consulted for discussion of the literature.

THE PROBLEM

The system to be treated analytically and approached experimentally consists of the steady laminar flow of a Newtonian fluid with constant physical properties. At the thermal entrance the wall temperature undergoes a step change in temperature to a value which may be below the freezing point of the liquid. Initial conditions for the flow are uniform temperature and parabolic velocity profile. Axial heat conduction, viscous energy dissipation, radiation, and free convection effects are regarded as negligible. This latter assumption is invoked in the analysis, but since the condition is approached only asymptotically in the experiment, allowance is made for free convection influence in the analysis of the data. Further conditions regarding the solidified layer are that the liquid–solid interface is smooth, the thickness is monotonically increasing with distance, and that the layer is pure and homogeneous.

The critical assumption regarding the flow in the heat transfer section which makes the problem readily amenable to solution in a straightforward manner is that the velocity profile remains parabolic in spite of the increasing ice layer thickness and decreasing channel size. Zerkle argues that the 'blunting' of the velocity profile due to the converging channel may be countered by the high liquid viscosity at the wall. With this assumption, the formulation is exactly the same as the classical Graetz problem. The only difference here is caused by the presence of the ice layer which continuously increases in the downstream direction and causes the flow to accelerate. However, in fully developed laminar flow the Nusselt number is uniquely a function of the Graetz number which can be written in the following way: $Gz = 4\,mc/\pi kx$. Thus at any position, one expects Nu to depend only on flow rate, m, and if this is preserved, Nu should remain constant in spite of a changing diameter. The problem then clearly independent of the ice layer, and Nu may be obtained as a function of Gz from any of several sources. Once the heat flux is determined, the ice layer thickness is calculated by simple conduction theory. Of major importance also is the pressure drop, but this calculation presents no problem using the channel size to determine the mean velocity, previous assumptions pertaining to the flow, and conventional laminar flow theory. However, the comparison of data with theory in this paper will make use of Zerkle's computed results instead of working through the solution as outlined above.

EXPERIMENTAL APPARATUS

The experimental system consists of water and refrigeration loops with instrumentation for the measurement of temperatures, pressures and water flow rate; it was designed and built to provide as closely as possible the conditions of the problem described in the previous section. The apparatus is shown schematically in Fig. 1.

The test section is a single pass counter-flow water to Freon-12 heat exchanger with the inner tube 22·3 in. long and 0·786 in. i.d. ($L/D = 28{\cdot}3$). This tube received water with uniform temperature and fully developed velocity profile from the inlet section. The test section is cooled by Freon evaporating as it passes in turbulent flow through the annular section formed by the exterior of the test section $\frac{3}{4}$ in. tube and an outer wall of $1\frac{1}{4}$ in. hard drawn copper tubing. Temperature of the test section wall is controlled by modulating the Freon evaporation pressure. The outlet mixing chamber is constructed from plexiglass which provides good thermal insulation and allows viewing of the outlet flow. As depicted on Fig. 1, outlet flow passes into the weighing tank, used in the flow rate determination, and from there returns to the reserve tank.

Pressure pickup points for the test section are located 3 in. upstream of the cold wall inlet in the calming section wall and in a $\frac{1}{8}$ in. dia. blunt pressure probe which extends axially into the test section from its downstream end. Pressure taps are 0·0135 in. dia. with holes on opposing sides to correct for any slight misalignment. In addition the probe supports a 30 ga. thermocouple extending about $\frac{1}{8}$ in. upstream into the flow. The thermocouple time constant is small enough to allow dynamic sensing of flow centerline temperature; flow turbulence is indicated when it exists.

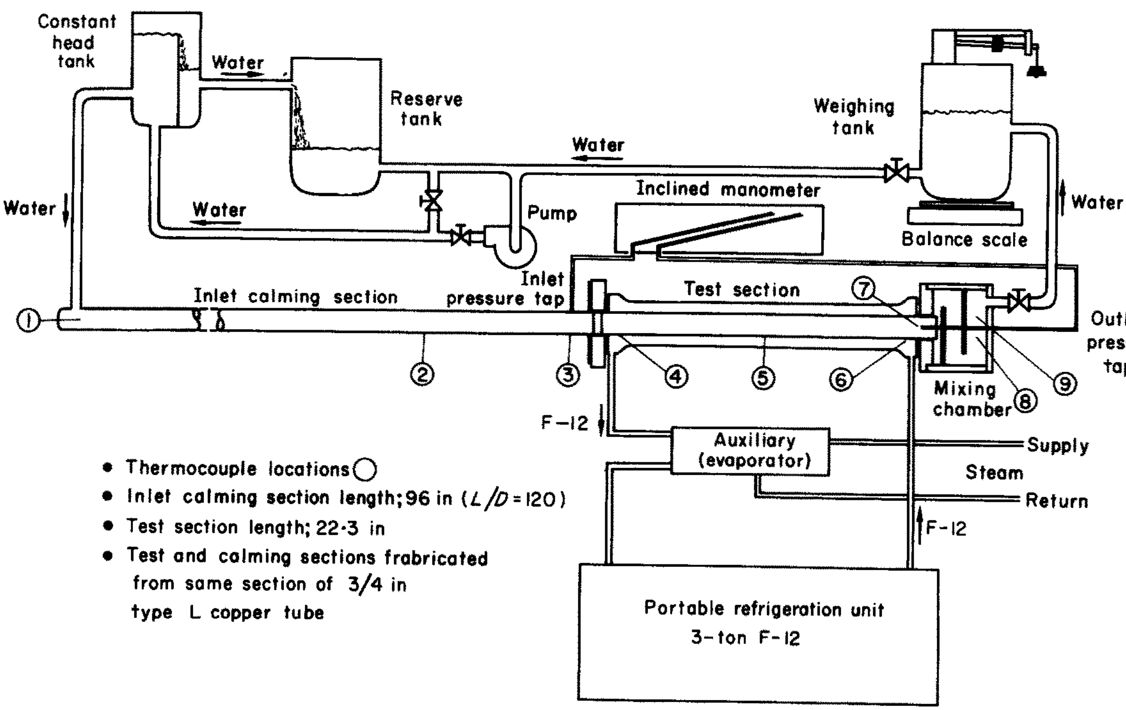

FIG. 1.

Table 1. Experimental data

Run no.	Gz	Z^*	Nu	q^*	m (lb/min)	T_0 (°F)	$\overline{T}_b$ (°F)	T_w (°F)	p^*
1	398	0·0101	13·81	0·128	3·34	69·5	67·2	30·1	4·9
2	364	0·01100	13·53	0·135	3·08	75·0	71·9	35·0	4·0
3	354	0·01135	13·88	0·145	2·98	71·0	68·1	23·3	10·0
4	179	0·02236	13·28	0·254	1·50	71·0	66·1	22·7	37·0
5	146	0·02741	17·07	0·374	1·22	71·0	65·0	23·3	52·8
6	255	0·01572	13·93	0·194	2·06	51·8	50·2	26·3	30·3
7	450	0·0089	15·54	0·128	3·71	60·0	58·5	25·9	8·3
8	378	0·01057	13·72	0·132	3·12	60·0	58·3	26·0	9·8
9	273	0·01464	12·97	0·170	2·25	60·0	57·8	26·4	16·3
10	200	0·02005	12·93	0·226	1·64	60·0	57·0	26·0	28·0
11	204	0·01957	14·88	0·250	1·72	72·5	67·8	26·8	15·4
12	168	0·02384	14·26	0·285	1·41	73·0	67·4	26·0	22·9
13	120	0·03340	15·24	0·400	1·00	72·0	64·2	25·5	42·0
14	131	0·03065	15·97	0·384	1·09	71·8	64·4	26·0	42·3
15	154	0·02594	16·48	0·336	1·29	71·6	15·2	26·0	31·2
16	224	0·01782	14·02	0·220	1·88	69·8	65·8	26·0	15·5
17	317	0·01264	14·07	0·161	2·65	68·5	65·7	26·2	5·8

EXPERIMENTAL RESULTS

Table 1 gives a list of the pertinent system variables and results. Nu and q^*, the dimensionless heat transfer rate, are both presented to allow direct comparison with other works in the discussion to follow. They are related by the equation

$$q^* = \frac{2\,Z^*\,Nu}{2 + Z^*\,Nu} \tag{1}$$

where Nu is based on the arithmetic mean temperature difference. The heat transfer rate is determined from the flow rate and bulk temperature change. Gz (or equivalently Z^*) is based on the measured mass flow rate and physical properties which are evaluated at the average bulk temperature. It should be noted that even though the wall temperature was below the freezing point for water in all tests reported in Table 1, 32°F was used in the calculation of Nu in all cases. The heat transfer results are presented in Fig. 2 as q^* vs. Z^*. Notice that the line marked "theory" represents the sought-after relationship and that all of the results are considerably above the curve. The pressure drop results are also presented in Table 1 and some are reproduced in Fig. 3 as p^* vs. Z^*.

DISCUSSION

Figure 2 shows the experimental results plotted as q^* vs. Z^* along with Zerkle's results and the analytic solution for laminar flow of a fluid having constant physical properties in an isothermal tube. It is clear that absolute verification of the model was not obtained since the results are from 25 per cent higher at low Z^* to 100 per cent higher at larger Z^* than the theoretical values. If a parabolic velocity profile

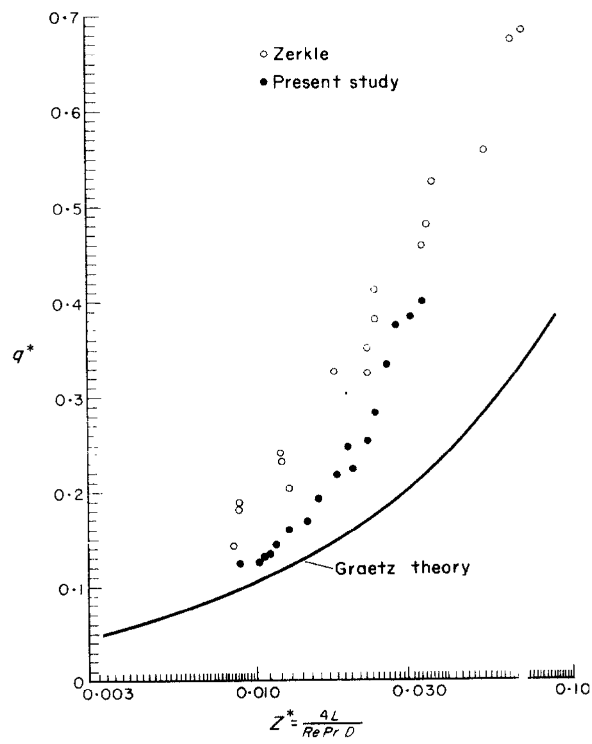

FIG. 2.

with constant transport properties had prevailed throughout the freezing section, agreement would have been obtained. It is also clear that substantially better agreement was obtained from the present investigation than by the previous work, and that the results are asymptotically approaching the theoretical curve at low Z^*. Smaller values of Z^* were unobtainable with the present equipment due to the occurrence of transition to turbulence.

Zerkle suggests that the difference between his experimental results and the Graetz solution is accountable by an empirical equation for combined free and forced convection due to Oliver [4]:

$$Nu = 1{\cdot}75 \left(\frac{\mu_w}{\mu_b}\right)^{0{\cdot}14} \left[F_1 \; Gz + 5{\cdot}6 \times 10^{-4} \left(Gr\, Pr \frac{L}{D}\right)^{0{\cdot}7}\right]^{\frac{1}{3}} \qquad (2)$$

where F_1 is a correction factor for the use of the arithmetic mean temperature difference. All physical properties are evaluated at the average bulk temperature, $\bar{T}_b$, using Ref. [5], and the temperature difference in the Grashof number is $\bar{T}_b - T_w$. The values of Nu predicted by equation (2) were calculated using only the flow rate, inlet temperature, and wall temperature from the experimental conditions. It is necessary to iterate equation (2) for F_1 and the outlet temperature. The error allowed for the solution was 0·1 per cent for Nu and 0·01°F for outlet temperature. Nu are interpreted as q^* and tabulated in Table 2. The error is also presented to indicate the accuracy of the method. The average error is 23 per cent, and the test results are always higher than the predicted values. Note that the wall temperature was taken appropriately to be 32°F, but that allowance was not made for the ice layer when using physical dimensions in Gr and in L/D. The inside tube diameter was always used due to the uncertainty in the evaluation of the ice layer thickness. An estimate of the maximum thickness yielded a value of about 0·005 ft. Since the tube diameter is 0·065 ft, the ice layer could have a non-negligible influence. The effect would, however, always serve to decrease the predicted value of Nu which is already lower than the experimental value.

Table 2. Predicted q^* and errors

Present work Run no.	q^* equation (2)	% error	Zerkle and Sunderland [3] Run no.	q^* equation (2)	% error
1	0·119	7·9	1	0·133	7·2
2	0·129	4·9	2	0·187	8·6
3	0·130	11·9	3	0·294	11·6
4	0·214	18·9	4	0·137	38·7
5	0·248	51·1	5	0·177	36·8
6	0·153	26·9	6	0·240	37·30
7	0·106	20·5	7	0·138	32·4
8	0·120	10·3	8	0·177	30·9
9	0·150	13·0	9	0·291	20·5
10	0·188	20·4	10	0·354	8·0
11	0·196	27·8	11	0·461	4·3
12	0·227	25·8	12	0·685	0·2
13	0·288	38·9	13	0·345	19·2
14	0·270	42·0	14	0·466	13·0
15	0·240	40·2	15	0·656	2·7
16	0·180	22·0	16	0·443	3·7
17	0·140	15·4	17	0·591	−5·4

Predictions of q^* using Oliver's correlation for Zerkle's test conditions are also tabulated in Table 2 along with the percentage errors. The average error is 16 per cent when considering all of his runs. Notice, however, that the average error for the first nine runs is 25 per cent when the test section had a length such that $L/D = 19$ but that there is only 6 per cent error in the last eight runs where $L/D = 53{\cdot}75$.

A more direct comparison with Oliver's correlation is shown in Fig. 3 where it can be seen that Zerkle's long tube tests fall on both sides of the correlation, and the scatter is within the range of data used in developing the equation. Note that the results shown in Fig. 3 are based on the average bulk temperatures from the experimental data. That the results from Zerkle's short tube and the present investigation ($L/D = 28{\cdot}6$) do not agree well with the correlation is understandable in view of the absence of data on short tubes in Oliver's work. Oliver warns against the use of his results in cases where L/D is 50 or less, and he offers the power of L/D as a provisional value which is subject to change with further research. The results of this investigation as well as Zerkle's work indicate that Oliver's correlation underestimates free convection effects by a considerable amount when $L/D < 50$, but the uncertainty of the liquid–solid interface diameters precludes accurate correlation of the present results with L/D.

The pressure drop results are tabulated in Table 1. The pressure drop is very sensitive to wall temperature by dependence on the ice layer thickness. Test section wall temperatures are normally only a few degrees below the freezing point, and this temperature drop determines the ice layer thickness. A small experimental error or wall temperature non-uniformity causes appreciable pressure drop variation. For example, typical conditions for a test were $T_0 = 65$°F: $T_w = 28{\cdot}5$°F, $Z^* = 0{\cdot}010$ such that $T_w^* = 0{\cdot}398$. Zerkle's plot of p^* vs. Z^* (Fig. 4) shows p^* to be 2·3. However, changing T_w by 1°F to 27·5°F changes p^* to 3·0, an increase of 30 per cent. p^* from runs 7–10 had wall temperatures very close together and produce a meaningful comparison to the analysis in Fig. 4. p^* based on the central T_w and the exit T_w are both shown. It is apparent that considerable latitude is available in the prediction of pressure drop. The actual wall temperature variation from the center to the exit represented in the four runs in Fig. 4 is about 6°F, but p^* varies by a factor of 4. The test data fall within the range, and are represented by an an average temperature between the center and the exit.

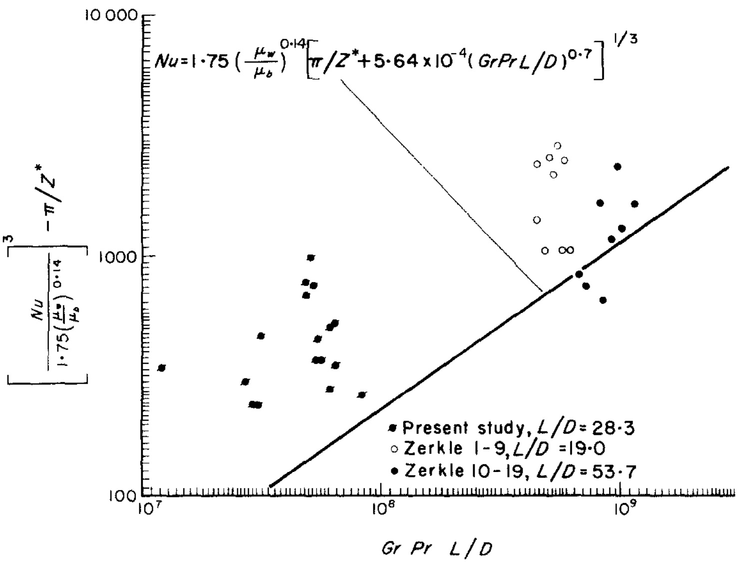

FIG. 3.

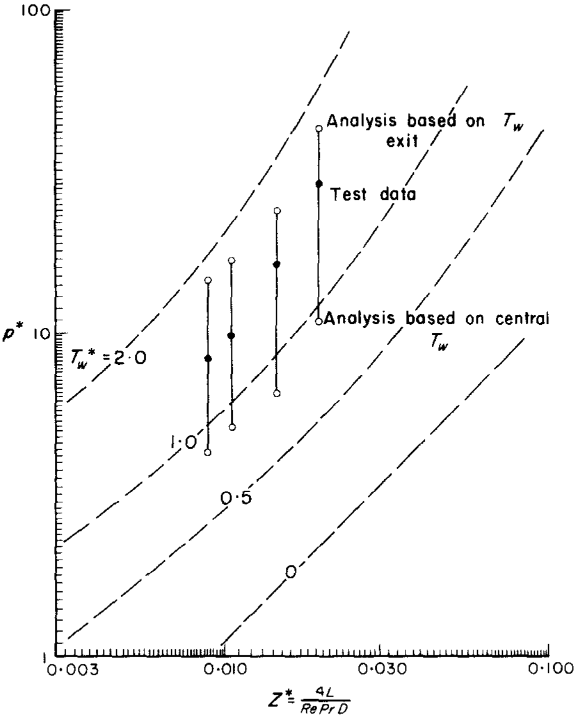

FIG. 4.

CONCLUSIONS

In conclusion, the following statements regarding the results can be made:

1. It was demonstrated in the limit that Zerkle's assumption of a parabolic velocity profile in the presence of an ice layer build up is valid when free convection is negligible.
2. Oliver's correlation of the influence of free convection with forced convection underestimates heat transfer when $L/D < 50$.
3. Pressure drop is extremely sensitive to wall temperature and considerable variation may result from only negligible temperature changes.

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